Ambipolar Diffusion Coefficient

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Ambipolar diffusion uses the two fluid equations to solve for the flux of electrons and ions in a partially ionized, low-temperature plasma, due to an electric field, a pressure gradient, and collisions with neutrals. Our two-fluid equations for the electrons and ions are

$$m_e n \frac{d\boldsymbol{u}_e}{dt} = -en\boldsymbol{E} - \boldsymbol{\nabla} P_e - \nu_{en} n m_e \boldsymbol{u}_e \tag{1}$$

$$m_i n \frac{d\boldsymbol{u}_i}{dt} = en\boldsymbol{E} - \boldsymbol{\nabla} P_i - \nu_{in} n m_i \boldsymbol{u}_i \tag{2}$$

The last term in each equation is a friction term, equivalent to $\boldsymbol{R}_{en}$ and $\boldsymbol{R}_{in}$ where the neutrals have zero net velocity.

We assume that we're in steady state and any velocities are small, so the LHS is approximately zero. We'll also assume the temperatures are constant, so $\boldsymbol{\nabla} P_e = T_e \boldsymbol{\nabla} n$ and $\boldsymbol{\nabla} P_i = T_i \boldsymbol{\nabla} n$. We can then assume the electrons and ions diffuse at the same rate, which gives us

$$0 = -en\boldsymbol{E} - T_e \boldsymbol{\nabla} n - \nu_{en} n m_e \boldsymbol{u} \tag{3}$$

$$0 = en\boldsymbol{E} - T_i \boldsymbol{\nabla} n - \nu_{in} n m_i \boldsymbol{u} \tag{4}$$

Adding equations (3) and (4) gives (eliminating the electric field)

$$0 = -(T_e + T_i)\boldsymbol{\nabla} n - (\nu_{en} m_e + \nu_{in} m_i) n\boldsymbol{u} \tag{5}$$

Solving for the flux of particles $\Gamma = n\boldsymbol{u}$, we have

$$\Gamma = -\frac{(T_e + T_i)}{\nu_{en} m_i + \nu_{in} m_i} \boldsymbol{\nabla} n \approx -\frac{T_e}{\nu_{in} m_i} \boldsymbol{\nabla} n \tag{6}$$

So the ambipolar diffusion coefficient is $D_{amb} = T_e / m_i \nu_{in}$.